

Nonabelian solutions in $\mathcal{N} = 4$, $D = 5$ gauged supergravity

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Abstract

We consider static, nonabelian solutions in $\mathcal{N} = 4$, $D = 5$ Romans' gauged supergravity model. Numerical arguments are presented for the existence of asymptotically anti-de Sitter configurations in the $\mathcal{N} = 4^+$ version of the theory, with a dilaton potential presenting a stationary point. Considering the version of the theory with a Liouville dilaton potential, we look for configurations with unusual topology. A new exact solution is presented, and a counterterm method is proposed to compute the mass and action.

1 Introduction

In the last few years, there has been increasing interest in solutions of various supergravity models with nonabelian matter, following the discovery by Chamseddine and Volkov [1] of a nontrivial monopole-type supersymmetric vacua in the context of the $\mathcal{N} = 4$ $D = 4$ Freedman-Schwarz gauged supergravity [2]. This is one of the few analytically known configurations involving both non-abelian gauge fields and gravity (for a general review of such solutions see [3, 4]). Its ten-dimensional lift was shown to represent 5-branes wrapped on a shrinking S^2 [5]. As discovered by Maldacena and Nuñez, this solution provides a holographic description for $\mathcal{N} = 1$, $D = 4$ super-Yang-Mills theory [6].

Chamseddine and Volkov looked also for five dimensional nonabelian configurations [7] in a version of $\mathcal{N} = 4$ Romans' gauged supergravity model [8] with a Liouville dilaton potential. The static spherically symmetric solution they found (although not in a closed form) possesses two unbroken supersymmetries and has been shown by Maldacena and Nastase to describe, after lifting to ten dimensions, the supergravity dual of an NS5-brane wrapped on S^3 with a twist that preserves only $\mathcal{N} = 1$ supersymmetry in 2+1 dimensions [9]. Both particle-like and black hole generalizations of the $D = 5$ Chamseddine-Volkov solution are discussed in [10] from a ten-dimensional perspective, where a background subtraction method to compute the mass and action of these configuration is also proposed. Although the dilaton potential $V(\phi)$ can be viewed as an effective negative, position-dependent cosmological term, these solutions do not have a simple asymptotic behaviour, the dilaton diverging at infinity.

However, as discussed in this paper, the situation is different for the $\mathcal{N} = 4^+$ version of the Romans model, with a dilaton potential consisting of the sum of two Liouville terms. In this case, the dilaton field approaches asymptotically a constant value ϕ_0 , which corresponds to an extremum of the potential such that $dV/d\phi|_{\phi_0} = 0$ and $V(\phi_0) < 0$. This makes possible the existence of both regular and black hole solutions approaching at infinity the anti-de Sitter (AdS) background. Topological black holes with nonabelian hair are found as well, in which case the three-sphere is replaced by a three-dimensional space of negative or vanishing curvature.

Considering next the case of the Romans' model with a Liouville dilaton potential, it is natural to ask if apart from black holes discussed in [10], whose horizon has spherical topology, there are also topological black holes with nonabelian fields. Such solutions are known to exist in an Abelian truncation of the theory (see e.g. [11]) and also in a four dimensional Einstein-Yang-Mills (EYM) system with negative cosmological constant [12]. However, we find that the inclusion of nonabelian fields leads to a pathological supergravity background: the factor multiplying the hyperbolic or flat surface gets negative for some finite values of the radial coordinate. The mass and action of the spherically symmetric solutions with a Liouville dilaton potential is computed by using a boundary counterterm method, the standard background subtraction results being recovered.

A general discussion of the $D = 5$ configurations admitting a translation along the fourth spatial coordinate is presented in Section 5. Two new exact solutions are found by uplifting topologically nontrivial configurations of the $\mathcal{N} = 4$ $D = 4$ Freedman-Schwarz model. We give our conclusions and remarks in the final section.

2 General framework

2.1 Action principle and field equations

The bosonic matter content of the Romans' gauged supergravity consists of gravity, a scalar ϕ , an $SU(2)$ Yang-Mills (YM) potential A_μ^I (with field strength $F_{\mu\nu}^I = \partial_\mu A_\nu^I - \partial_\nu A_\mu^I + g_2 \epsilon^{IJK} A_\mu^J A_\nu^K$), an abelian potential W_μ ($f_{\mu\nu}$ being the corresponding field strength), and a pair of two-form fields. These two form fields can consistently be set to zero, which yields the bosonic part of the action

$$I_5 = \frac{1}{4\pi} \int_{\mathcal{M}} d^5x \sqrt{-g} \left(\frac{1}{4} \mathcal{R} - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{4} e^{2a\phi} F_{\mu\nu}^I F^{I\mu\nu} - \frac{1}{4} e^{-4a\phi} f_{\mu\nu} f^{\mu\nu} \right. \\ \left. - \frac{1}{4\sqrt{-g}} \epsilon^{\mu\nu\rho\sigma\tau} F_{\mu\nu}^I F_{\sigma\tau}^I W_\tau - V(\phi) \right) - \frac{1}{8\pi} \int_{\partial\mathcal{M}} d^4x \sqrt{-h} K, \quad (1)$$

where $a = \sqrt{2/3}$. Here

$$V(\phi) = -\frac{g_2^2}{8} \left(e^{-2a\phi} + 2\sqrt{2} \frac{g_1}{g_2} e^{a\phi} \right) \quad (2)$$

is the dilaton potential, g_1 being the $U(1)$ gauge coupling constant. The last term in (1) is the Hawking-Gibbons surface term, necessary to ensure that the Euler-Lagrange variation is well defined, where K is the trace of the extrinsic curvature for the boundary $\partial\mathcal{M}$ and h is the induced metric of the boundary.

As discussed in [8], the theory has three canonical forms, corresponding to different choices of the gauge coupling constants in (2). The case $g_2 = 0$ corresponds to $\mathcal{N} = 4^0$ theory, where the $SU(2) \times U(1)$ symmetry is replaced by the abelian group $U(1)^4$; there is also a $\mathcal{N} = 4^+$ version in which $g_2 = g_1 \sqrt{2}$, and $\mathcal{N} = 4^-$ with $g_2 = -g_1 \sqrt{2}$. Also, for this truncation of the theory with vanishing two forms, one can take $g_1 = 0$ and find another distinct case. Note that for a nonvanishing g_2 , one can set its value to one, by using a suitable rescaling of the field.

The field equations are obtained by varying the action (1) with respect to the field variables $g_{\mu\nu}$, A_μ^I , W_μ and ϕ

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 2 T_{\mu\nu}, \\ \nabla^2 \phi - \frac{a}{2} e^{2a\phi} F_{\mu\nu}^I F^{I\mu\nu} + a e^{-4a\phi} f_{\mu\nu} f^{\mu\nu} - \frac{\partial V}{\partial \phi} = 0, \\ D_\nu (e^{-4a\phi} f^{\mu\nu}) - \frac{1}{4\sqrt{-g}} \epsilon^{\mu\nu\rho\sigma\tau} F_{\nu\rho}^I F_{\sigma\tau}^I = 0, \\ D_\nu (e^{2a\phi} F^{I\mu\nu}) - \frac{1}{2\sqrt{-g}} \epsilon^{\mu\nu\rho\sigma\tau} F_{\nu\rho}^I f_{\sigma\tau} = 0, \quad (3)$$

where the energy-momentum tensor is defined by

$$T_{\mu\nu} = \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} g_{\mu\nu} \partial_\sigma \phi \partial^\sigma \phi - g_{\mu\nu} V(\phi) \\ + e^{2a\phi} (F_{\mu\rho}^I F_{\nu\sigma}^I g^{\rho\sigma} - \frac{1}{4} g_{\mu\nu} F_{\rho\sigma}^I F^{I\rho\sigma}) + e^{-4a\phi} (f_{\mu\rho} f_{\nu\sigma} g^{\rho\sigma} - \frac{1}{4} g_{\mu\nu} f_{\rho\sigma} f^{\rho\sigma}). \quad (4)$$

2.2 The ansatz

Restricting to static solutions, we consider a general metric ansatz on the form

$$ds^2 = A^2(r)dr^2 + B^2(r)d\Sigma_{3,k}^2 - C^2(r)dt^2, \quad (5)$$

where $d\Sigma_{3,k}^2 = d\psi^2 + f_k^2(\psi)d\Omega_2^2$ denotes the line element of a three-dimensional space with constant curvature ($d\Omega^2 = d\theta^2 + \sin^2\theta d\varphi^2$ being the round metric of S^2). The discrete parameter k takes the values 1, 0 and -1 and implies the form of the function $f_k(\psi)$

$$f_k(\psi) = \begin{cases} \sin \psi, & \text{for } k = 1 \\ \psi, & \text{for } k = 0 \\ \sinh \psi, & \text{for } k = -1. \end{cases} \quad (6)$$

When $k = 1$, the hypersurface $\Sigma_{3,1}$ represents a 3-sphere; for $k = -1$, it is a 3-dimensional negative constant curvature space and it could be a closed hypersurface with arbitrarily high genus under appropriate identifications. For $k = 0$, $\Sigma_{3,0}$ is a three-dimensional Euclidean space.

For the matter fields ansatz, we start by choosing a purely-electric abelian field ansatz

$$f = f_{rt}(r)dt \wedge dr, \quad (7)$$

the dilaton field being also a function only on the coordinate r .

The computation of the most general expression for A_μ^I compatible with the symmetries of the line-element (5) is a straightforward generalization of the $k = 1$ case discussed in Appendix A of [13]. Applying the standard rules for calculating the gauge potentials for any spacetime group [14, 15], and taking $A_t^I = 0$ *i.e.* no dyons, one finds the ansatz (with τ_a the Pauli spin matrices)

$$A = \frac{1}{2} \left\{ \tau_3(\omega(r)d\psi + \cos\theta d\varphi) - \frac{df_k(\psi)}{d\psi}(\tau_2 d\theta + \tau_1 \sin\theta d\varphi) + \omega(r)f_k(\psi)(\tau_1 d\theta - \tau_2 \sin\theta d\varphi) \right\}, \quad (8)$$

the corresponding YM curvature being

$$F = \frac{1}{2} \left(\omega' \tau_3 dr \wedge d\psi + \omega' f_k \tau_1 dr \wedge d\theta - f_k \omega' \tau_2 dr \wedge d\varphi + (k - w^2) f_k \tau_2 d\psi \wedge d\theta \right. \\ \left. + (k - w^2) f_k \sin\theta \tau_1 d\psi \wedge d\varphi + (w^2 - k) f_k^2 \sin\theta \tau_3 d\theta \wedge d\varphi \right), \quad (9)$$

where a prime denotes a derivative with respect to r .

For a purely magnetic YM ansatz, the equation for the Abelian field

$$\nabla_\nu (e^{-4a\phi} f^{\nu\mu}) = \frac{1}{4\sqrt{-g}} \epsilon^{\mu\nu\rho\sigma\tau} F_{\nu\rho}^I F_{\sigma\tau}^I \quad (10)$$

have a total derivative structure, implying after the integration the simple expression for f_{tr}

$$f_{tr}(r) = \frac{e^{4a\phi}}{\sqrt{-g}} (2\omega^2 - 6k\omega + c) A^2(r) C^2(r), \quad (11)$$

where c is a constant of integration.

3 $\mathcal{N} = 4^+$ solutions

The scalar potential for the $\mathcal{N} = 4^+$ model $V(\phi) = -(e^{-2a\phi} + 2e^{a\phi})/8$ has exactly one extremum at $\phi = 0$, corresponding to an the effective cosmological constant

$$\Lambda_{eff} = 2V(0) = -\frac{3}{4}. \quad (12)$$

As discussed by Romans [8], the maximally symmetric AdS_5 spacetime is a solution of the theory for $\phi \equiv 0$ and pure gauge fields, preserving the full $\mathcal{N} = 4$ supersymmetry.

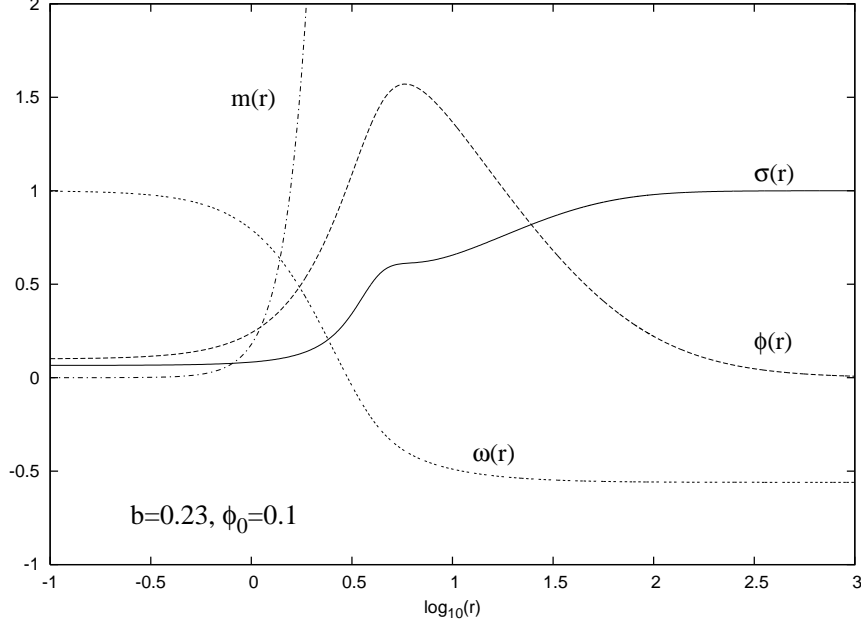


Figure 1. The gauge function ω , the dilaton ϕ and the metric functions $m(r)$, $\sigma(r)$ are shown as function of the coordinate r for a typical spherically symmetric regular solution.

Here we look for configurations with nonvanishing matter fields, approaching asymptotically the AdS background.

In deriving numerical solutions, it turns out to be convinient to use the following parametrization of the line element (5)

$$A^2(r) = \frac{1}{H(r)}, \quad B^2(r) = r^2, \quad C^2(r) = H(r)\sigma^2(r) \quad (13)$$

The existence in the asymptotic region of an effective negative cosmological constant suggest to use the following form of the metric function $H(r)$

$$H(r) = k - \frac{4m(r)}{3r^2} + \frac{r^2}{\ell^2}, \quad (14)$$

(with $\Lambda_{eff} = -6/\ell^2$ *i.e.* the characteristic length scale $\ell^2 = 8$), the function $m(r)$ being related in the usual approach to the local mass-energy density, up to some numerical factor.

Inserting this ansatz into the action (1), the field equations reduce to

$$\begin{aligned} m' &= \frac{1}{2}\phi'^2 H r^3 + \frac{3}{2}e^{2a\phi} r (H\omega'^2 + \frac{(\omega^2 - k)^2}{r^2}) + \frac{1}{2}e^{4a\phi} \frac{(2\omega^3 - 6k\omega + c)^2}{r^3}, \\ \frac{\sigma'}{\sigma} &= \frac{2}{3}r\phi'^2 + \frac{2}{r}e^{2a\phi}\omega'^2, \\ (e^{2a\phi}\sigma r H \omega')' &= 2e^{2a\phi}\sigma \frac{\omega}{r}(\omega^2 - k) + \frac{1}{3}e^{4a\phi} \frac{6(2\omega^3 - 6k\omega + c)(\omega^2 - k)}{r^3}, \\ (\sigma r^3 H \phi')' &= 3ae^{2a\phi}\sigma r (H\omega'^2 + \frac{\omega^2 - k)^2}{r^2}) + 2ae^{2a\phi}\sigma \frac{(2\omega^3 - 6k\omega + c)^2}{r^3} + \frac{a}{4}(e^{-2a\phi} - e^{a\phi})r^3\sigma, \end{aligned} \quad (15)$$

while the U(1) gauge field is given by (11).

These equations present both globally regular and black hole solutions.

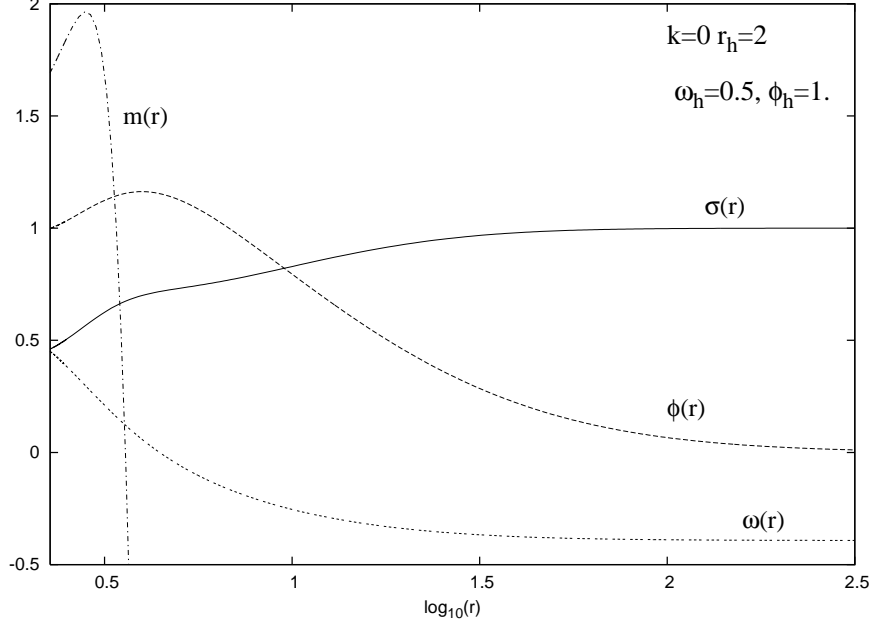


Figure 2. The gauge function ω , the dilaton ϕ and the metric functions $m(r)$, $\sigma(r)$ are shown as function of the coordinate r for a typical $k = 0$ topological black hole solution.

Regular configurations exist for $k = 1$, $c = 4$ only, and have the following expansion near the origin

$$\begin{aligned} m(r) &= (3b^2e^{2a\phi_0} - \frac{1}{32}(e^{-2a\phi_0} + 2e^{-2a\phi_0} - 3))r^4 + O(r^5), \quad \sigma(r) = \sigma_0(1 + 4b^2e^{2a\phi_0})r^2 + O(r^2), \\ \phi(r) &= \phi_0 + \left(3ab^2e^{2a\phi_0} + \frac{a}{16}(e^{-2a\phi_0} - e^{-2a\phi_0})\right)r^2 + O(r^4), \quad \omega(r) = 1 - br^2 + O(r^4), \end{aligned} \quad (16)$$

with b , ϕ_0 , σ_0 real constants.

We are also interested in black hole solutions having a regular event horizon at $r = r_h > 0$. The field equations implies the following behaviour as $r \rightarrow r_h$ in terms of three parameters $(\phi_h, \sigma_h, \omega_h)$

$$\begin{aligned} m(r) &= \frac{3}{4}r_h^2(k + \frac{r_h^2}{\ell^2}) + m_1(r - r_h) + O(r - r_h)^2, \quad \sigma(r) = \sigma_h + \sigma_1(r - r_h) + O(r - r_h)^2, \\ \phi(r) &= \phi_h + \phi_1(r - r_h) + O(r - r_h)^2, \quad \omega(r) = \omega_h + \omega_1(r - r_h) + O(r - r_h)^2, \end{aligned} \quad (17)$$

where we defined

$$\begin{aligned} m_1 &= \frac{1}{2r_h^3} \left(e^{4a\phi_h} (2\omega_h^3 - 6k\omega_h + c)^2 + r_h^2 (3e^{2a\phi_h} (\omega_h^2 - k)^2 - \frac{1}{4}r_h^4 (e^{-2a\phi_h} + 2e^{-2a\phi_h} - 3)) \right), \\ \omega_1 &= \frac{6r_h^2\omega_h(\omega_h^2 - k) + 6e^{2a\phi_h}(2\omega_h^3 - 6k\omega_h + c)(\omega_h^2 - k)}{2r_h^2(-2m_1 + 3kr_h + 6r_h^3/\ell^2)}, \\ \phi_1 &= \frac{3(2ae^{4a\phi_h}(2\omega_h^3 - 6k\omega_h + c)^2 + r_h^2(3ae^{2a\phi_h}(\omega_h^2 - k)^2 + r_h^4V'(\phi_h))}{2r_h^4(-2m_1 + 3kr_h + 6r_h^3/\ell^2)}. \end{aligned} \quad (18)$$

Note that since the equations (15) are invariant under the transformation $\omega \rightarrow -\omega$ one can set $\omega(0) > 0$, $\omega(r_h) > 0$ without loss of generality.

Using the above initial conditions, the equations (15) were integrated for a large set of b , ϕ_0 (ω_h , ϕ_h respectively) and several values of r_h . For all solutions presented here we set $c = 4$, although we studied black holes with other values of c also. The overall picture we find combines features of the pure five dimensional EYM- Λ system discussed in [13] and the four dimensional EYM-dilaton solutions with a potential approaching a constant negative value at infinity, considered in [16].

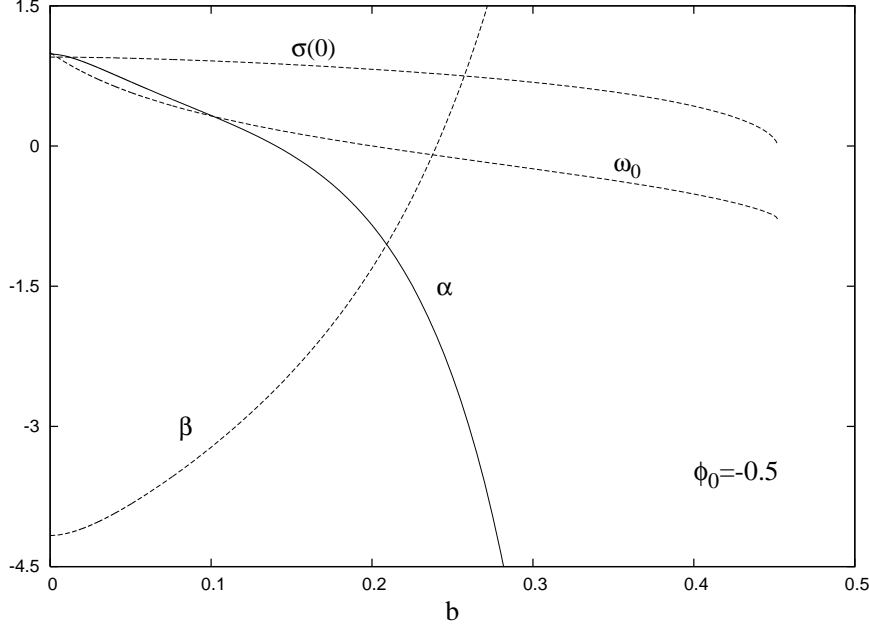


Figure 3. The asymptotic parameters ω_0 , α , β , and the value of the metric function $\sigma(r)$ at the origin are shown as a function of b for spherically symmetric regular solutions with $\phi_0 = -0.5$. For $b = 0$ one finds $\alpha \simeq 0.984$, $\beta \simeq -4.166$ and $\sigma(0) \simeq 0.951$.

A continuum of monopole solutions is obtained for compact intervals of the initial parameters. The gauge field interpolates monotonically between the initial value at the $r = r_i$ (with $r_i = 0$ or r_h) and some asymptotic value ω_0 . The value at the origin/event horizon of the metric function $\sigma(r)$ (which is a result of the numerical integration) decreases along these intervals and, at some stage a singularity appears, corresponding to $\sigma(r_i) \rightarrow 0$. Typical configurations are presented in Figure 1 (a spherically symmetric regular solution) and Figure 2 (a $k = 0$ black hole solution).

For all configurations we have studied, the function $m(r)$ diverges as $r \rightarrow \infty$. There are two distinct sources of this behavior. Considering first the dilaton sector, we note that the scalar field mass μ is given by

$$\mu^2 = V''(0) = -\frac{1}{2},$$

which saturates the Breitenlohner-Freedman bound [17]. Thus, the scalar field behaves asymptotically as

$$\phi(r) = \frac{\alpha}{r^2} + \frac{\beta \log r}{r^2} + \dots, \quad (19)$$

where α , β are real constants. For such solutions, due to the back reaction of the scalar field, the usual ADM mass diverges logarithmically with r as $r \rightarrow \infty$ [18, 19].

There is also a second logarithmic divergence of the function $m(r)$ associated with the nonabelian gauge field $\omega(r)$. For generic solutions, $\omega^2 \rightarrow \omega_0^2 \neq k$ as $r \rightarrow \infty$, which from (15) implies that asymptotically $m(r) \sim (\omega_0^2 - k)^2 \log r$. A finite contribution to the total mass is found for $\omega_0^2 = k$ only. However, we failed to find such solutions and it seems that, similar to the EYM-A case [13], only a pure gauge configuration has this asymptotics.

We remark that for any value of $\phi(r_i)$ it is possible to find a initial value of the gauge field such that $\beta = 0$, which removes the divergencies associated with the scalar field. Also, as seen in Figure 3 for a set of regular configurations, the solution with $b = 0$ (*i.e.* $\omega(r) \equiv 1$) and a nonzero ϕ_0 is not the vacuum AdS spacetime. Thus, for these asymptotics there are globally regular solutions even without a nonabelian field. One finds also black holes with scalar hair without a gauge field. Similar solutions have been found

in other models with a dilaton field possessing a nontrivial potential approaching a constant negative value at infinity (see e.g. [16, 18]).

A systematic analysis reveals the following expansion of the solutions at large r :

$$\begin{aligned} m(r) &= M + \frac{1}{16}\beta(\beta - 4\alpha)\log r - \frac{\beta^2}{8}\log^2 r + \frac{3}{2}(\omega_0^2 - k)^2\log r + \dots, \\ \log \sigma(r) &= -\frac{1}{3}\beta(4\alpha - \beta + 2\beta\log r)\frac{\log r}{r^4} - (\beta^2 - 4\alpha\beta + 8\alpha^2)\frac{1}{2r^4} + \dots, \\ \omega(r) &= \omega_0 - \ell^2\omega_0(\omega_0^2 - k)\frac{\log r}{r^2} + \frac{c_1}{r^2} + \dots, \quad \phi(r) = \frac{\alpha}{r^2} + \beta\frac{\log r}{r^2} + \dots, \end{aligned} \quad (20)$$

where c_1 is a constant. Note that the asymptotic metric still preserves the usual $\text{SO}(4,2)$ symmetry, and the spacetime is still asymptotically AdS, despite the diverging $m(r)$.

We close this section remarking that in the absence of a gauge field, it is possible to define a total mass by using the approach in [18, 19]. By employing an Hamiltonian method, the divergencies from the gravity and scalar parts cancel out, yielding a finite total charge. It would be interesting to generalize this approach in the presence of a nonabelian gauge field in the bulk and to define a finite mass and action for these configurations, too.

4 $g_1 = 0$ solutions

The properties of the solutions are very different if we set $g_1 = 0$ in (2), which leads to a Liouville-type dilaton potential

$$V(\phi) = -\frac{1}{8}e^{-2a\phi}. \quad (21)$$

In this case we found convenient to use the following parametrization of the metric ansatz (5)

$$A(r) = e^{a\phi(r)+Y(r)-X(r)}, \quad B(r) = e^{a\phi(r)}R(r), \quad C(r) = e^{a\phi(r)+X(r)}, \quad (22)$$

which yields the reduced action

$$\begin{aligned} L &= \frac{3}{2}e^{3a\phi+2X-Y}(R'^2 + 3aR\phi'R' + aR^2\phi'X' + RR'X' + R^2\phi'^2 + ke^{-2X+2Y}) \\ &\quad - \frac{3}{2}e^{3a\phi+Y}(e^{2X-2Y}R\omega'^2 + \frac{\omega^2 - k^2}{R}) - \frac{1}{2}e^{3a\phi+Y}\frac{(2\omega^3 - 6k\omega + c)^2}{R^3} + \frac{1}{8}e^{3a\phi+Y}R^3. \end{aligned} \quad (23)$$

We remark that (23) allows for the reparametrization $r \rightarrow \tilde{r}(r)$ which is unbroken by our ansatz.

In this way, we find that the field equations are (here we fix the metric gauge by setting $Y = 0$ and define $e^{2X} = \nu$)

$$\begin{aligned} R'' &= -\frac{2(2\omega^3 - 6k\omega + c)^2}{\nu R^5} + \frac{2k}{\nu R} - \frac{4}{\nu R^3}(\omega^2 - k)^2 - \frac{\nu'R'}{\nu} - \frac{2R'^2}{R} - \frac{2\omega'^2}{R} - \sqrt{6}R'\phi', \\ (e^{3a\phi}R\nu\omega')' &= e^{3a\phi}(\omega^2 - k)\left(\frac{2\omega}{R} + \frac{3(2\omega^3 - 6k\omega + c)}{2R^3}\right), \\ R'' + \frac{(k - \omega^2)^2}{\nu R^3} - \frac{k}{\nu R} + R'\left(\frac{\nu'}{\nu} + 2\phi'\right) + \frac{R'^2}{R} + \frac{\omega'^2}{R} &= 0, \\ \phi'' - \frac{(k - \omega^2)^2}{\nu R^4} + \frac{k}{\nu R^2} - \frac{\nu'R'}{\nu R} - \frac{R'^2}{R^2} - \frac{2R'\phi'}{R} &= 0, \\ X' - \alpha e^{Y-X-2s} &= 0, \end{aligned} \quad (24)$$

where α is an integration constant. In the "extremal" case $\alpha = 0$ we can set $X = 0$, without loss of generality. For $\alpha \neq 0$, we find black hole solutions with a nontrivial $\nu = e^{2X}$ metric function.

4.1 BPS configurations

As discussed in [7], [10], for $\alpha = 0$, $k = 1$ the system (24) presents configurations preserving some amount of supersymmetry and solving first order Bogomol'nyi equations.

To find such solutions for any value of k , it is convenient to introduce the new variables (also, to conform with other results in literature, we note here $c = 4\kappa$)

$$\phi = \sqrt{\frac{2}{3}}s - \sqrt{\frac{3}{2}}g - \sqrt{\frac{1}{6}}X, \quad R = e^g. \quad (25)$$

With this choice, the lagrangian (23) becomes

$$\begin{aligned} L = e^{X+2s} \left(\frac{2}{3}s'^2 - \frac{1}{2}g'^2 - \frac{X'^2}{3} \right) - e^{X+2s-2g}\omega'^2 - e^{-X+2s-4g}(\omega^2 - k)^2 \\ - \frac{1}{3}(2\kappa - 3k\omega + \omega^3)^2 e^{-X+2s-6g} + ke^{-X+Y+2s-2g} - \frac{1}{12}e^{-X+2s}, \end{aligned} \quad (26)$$

which can be written in the form

$$L = G_{ik}(y) \frac{dy^i}{dr} \frac{dy^k}{dr} - U(y), \quad (27)$$

where $y^i = (s, g, w)$ and $G_{ik} = e^{2s} \text{diag}(\frac{2}{3}, -\frac{1}{2}, -e^{-2g})$. The potential U can be represented as

$$U = -G^{ik} \frac{\partial W}{\partial y^i} \frac{\partial W}{\partial y^k}, \quad (28)$$

where the superpotential W has the expression

$$W = \frac{1}{6}e^{-g+2s} \sqrt{\frac{1}{2}e^{2g} - 6(\omega^2 - k) + 18e^{-2g}(\omega^2 - k)^2 - (e^{-2g}(2\omega^3 - 6k\omega + 4\kappa) - 3\omega)^2}. \quad (29)$$

As a result, we find the first order Bogomol'nyi equations $dy^i/dr = G^{ik} \partial W / \partial y^k$

$$\begin{aligned} s' &= \frac{1}{2\sqrt{2}}e^{-g+Y} \sqrt{(e^{2g} + 6(2k + \omega^2) + 8e^{-4g}(2\kappa - 3k\omega + \omega^3)^2 + 12e^{2g}(w^4 - 4\omega\kappa + 3k^2))}, \\ \omega' &= \frac{e^{-g+Y}(-e^{4g}\omega + 4e^{2g}(\kappa - \omega^3) + 4(k - \omega^2)(2\kappa - 3k\omega + \omega^3))}{\sqrt{2}\sqrt{e^{6g} + 6e^{4g}(2k + \omega^2) + 8(2\kappa - 3k\omega + \omega^3)^2 + 12e^{2g}(3k^2 - 4\kappa\omega + \omega^4)}}, \\ g' &= \frac{e^{2g}(-e^{4g}\omega + 4e^{2g}(\kappa - \omega^3) + 4(k - \omega^2)(2\kappa - 3k\omega + \omega^3))}{\sqrt{2}\sqrt{(e^{4g}(2k + \omega^2) + 4(2\kappa - 3k\omega + \omega^3)^2 + 4e^{2g}(3k^2 - 4\kappa\omega + \omega^4))}}, \end{aligned} \quad (30)$$

which solve also the second-order system. It can be proven that, after a suitable redefinition, these are the equations derived in [3] for $k = 1$, by using a Killing spinor approach.

Unfortunately, no exact solution of these equations can be found in the general case, except for the special values ($k = 0$, $\kappa = 0$). For a gauge choice $Y = g$, we find the the new exact solution of the Romans model

$$\begin{aligned} ds^2 &= e^{2\beta r/3+4s_0/3}(2(\beta^2 - e^{-2\gamma_0(r-r_0)}))^{1/3} \left(-\frac{dt^2}{2(\beta^2 - e^{-2\gamma_0(r-r_0)})} + dr^2 + d\Omega_{0,3}^2 \right), \\ w(r) &= e^{-\beta(r-r_0)}, \quad e^{2a\phi} = \frac{e^{4\beta r/3+4s_0/3}}{(2(\beta^2 - e^{-2\gamma_0(r-r_0)}))^{2/3}}, \end{aligned} \quad (31)$$

where s_0 , β , γ_0 , r_0 are arbitrary real constants.

A direct computation reveals that the above line element presents a curvature singularity for a finite value of the radial coordinate, $r = r^*$ (with $e^{-2\gamma_0(r^*-r_0)} = \beta^2$). This singularity appear to be repulsive:

no timelike geodesic hits it, though a radial null geodesic can. Thus our solution violate the criterion of [20] because g_{tt} in the Einstein frame is unbounded at the singularity and thus they cannot accurately describe the IR dynamics of a dual gauge theory.

Of course, in other cases, the equations (30) can be solved numerically. However, a direct inspection of the above relations reveal that the absence of solutions with a regular origin for $k \neq 1$ is a generic property of $k = 0, -1$ BPS solutions (we call regular origin the point $r = r_0$, where the function $R(r)$ vanishes but all curvature invariants are bounded (without loss of generality we can set $r_0=0$)).

It can be proven that this is a generic feature of all $k \neq 1$ configurations. Considering solutions of the second order equations (24), one finds that it is also not possible to take a consistent set of boundary conditions at the origin without introducing a curvature singularity. This fact has to be attributed to the particular form of the potential term $V_{YM} = e^{2a\phi}(\omega^2 - k)^2/(2R)$ in the reduced lagrangean of the system.

Solutions with regular origin may exist for $k = 1, \kappa = 1$ only and are parametrized by the value of the parameter b appearing in the expansion of $\omega(r)$ at the origin $\omega(r) = 1 - br^2 + O(r^4)$. As discussed in [10], globally regular solutions extending to infinity (*i.e.* an unbounded $R(r)$) exists for $0 < b < 1$ only, the BPS solution found in [7] corresponding to $b = 1/3$. For $b \geq 1$, the function $R(r)$ goes to zero again at some finite value of r .

4.2 Black hole solutions

A natural way to deal with the type of singularities we have found for $k = 0, -1$ is to hide them inside an event horizon. To implement the black hole interpretation we restrict the parameters so that the metric describes the exterior of a black hole with a non-degenerate horizon. That implies the existence of a point $r = r_h$ where $e^{2X} = \nu$ vanishes, while all other functions are finite and differentiable. Without loss of generality we can set $r_h = 0$.

The field equations (24) give the following expansion near the event horizon

$$\begin{aligned} R(r) &= R_h + R_1 r + O(r^2), \quad \phi(r) = \phi_h + \phi_1 r + O(r^2), \quad \nu(r) = \alpha \frac{e^{-3a\phi_h}}{R_h^3} r + O(r^2), \\ \omega(r) &= w_h + \frac{2e^{3a\phi_h}}{3\alpha R_h} (\omega^2 - k)(6R_h^2 \omega_h + 2\omega_h^3 - 6k\omega_h + c)r + O(r^2), \end{aligned} \quad (32)$$

where

$$\begin{aligned} \phi_1 &= \frac{ae^{3a\phi_h}}{8\alpha R_h^3} \left(R_h^6 - 12kR_h^4 + 36R_h^2(\omega^2 - k)^2 + 20(2\omega_h^3 - 6k\omega_h + c) \right), \\ R_1 &= \frac{e^{3a\phi_h}}{2\alpha R_h^2} \left(R_h^2(kR_h^2 + (\omega_h^2 - k)^2 + (2\omega_h^3 - 6k\omega_h + c)^2 - 12e^{-3a\phi_h}\alpha a\phi_1 R_h) \right). \end{aligned}$$

The solutions present three free parameters: the value of the dilaton at the horizon ϕ_h , the event horizon radius R_h and the value of the gauge potential at the horizon ω_h . For any k , one can set $\alpha = 1$ without loss of generality, since this value can be obtained by a global rescaling of the line element (5).

Using the initial conditions on the event horizon (32), the equations (24) were integrated for a range of values of ϕ_h , R_h and varying ω_h . The numerical analysis shows the existence of a continuum of solutions for every value of $(k, R_h, \omega_h, \phi_h)$. Also, for every choice of ϕ_h and a given (k, R_h, ω_h) , we find qualitatively similar solutions (different values of ϕ_h lead to global rescalings of the solutions).

The results we found for $k = 1$ are similar to those derived in [10] from a ten-dimensional point of view. One can show that in this case ω_h is restricted to $|\omega_h| \leq 1$, while the value of c is not restricted. Spherically symmetric black hole solutions are found for any set of values (ϕ_h, R_h, ω_h) . For $R_h^2 + \omega_h^2 > 1$, $R(r) \rightarrow \infty$ as $r \rightarrow \infty$, while for $R_h^2 + \omega_h^2 < 1$, as numerically found in [10], the asymptotic is different: $R(r)$ vanishes at some finite value of r , where there is a curvature singularity.

Unfortunately, the situation for a nonspherically symmetric event horizon resembles this last case. For each set (R_h, w_h, ϕ_h) we find a solution living in the interval $r \in [0, r^*[$, where r^* has a finite value. For fixed R_h, ϕ_h , the value of r^* decreases when increasing ω_h . A typical $k = -1$ solution is presented in Figure 4 for $c = 1, \alpha = 1$.

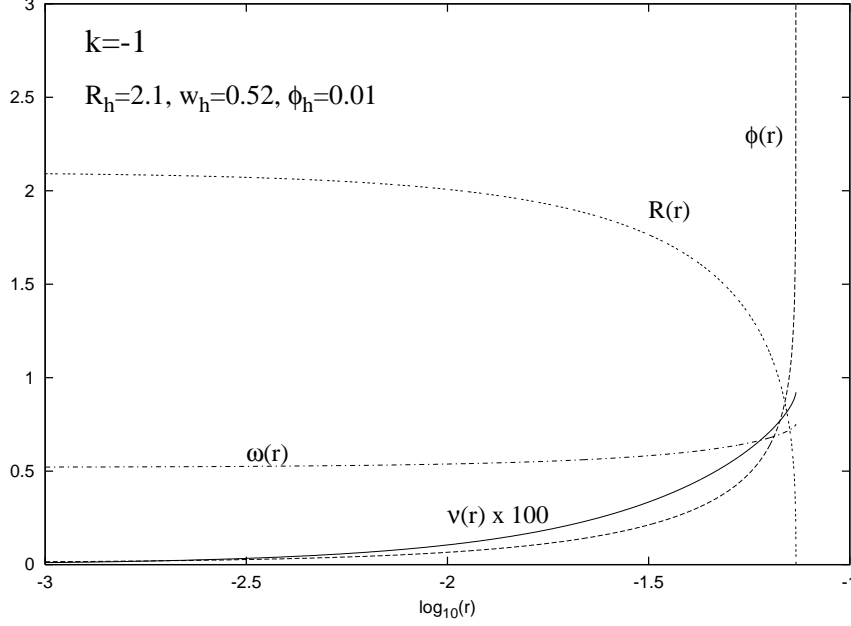


Figure 4. The gauge function ω , the dilaton ϕ and the metric functions R , $\nu = e^{2X}$ are shown as function of the coordinate r for a typical $k = -1$ solution with an event horizon at $R_h = 2.1$.

This fact can be understood by noticing that, for $k \neq 1$, the boundary conditions (32) imply that $R(r)$ is a strictly decreasing function near the event horizon (*i.e.* $R'(r_h) < 0$). In all cases we studied, $R(r)$ keeps decreasing for increasing r , vanishing at r^* where there is a curvature singularity. For $r > r^*$, a wrong signature spacetime is found.

Therefore, it seems that, for a Liouville dilaton potential, all nonabelian configurations with nonspherical topology present some pathological features. Given the presence of the naked singularities, the physical significance of these solutions is not obvious. A similar property has been noticed in [21] for $k = 0, -1$ solutions of $\mathcal{N} = 4$ $D = 4$ Freedman-Schwarz gauged supergravity model which possesses also a Liouville dilaton potential.

4.3 A counterterm proposal

In this section we'll concentrate on the computation of mass and action of the generic $k = 1$ spherically symmetric solutions for which $R(r) \rightarrow \infty$ as $r \rightarrow \infty$. We start by presenting the asymptotic expression of these solution [10], which is shared by both globally regular and black hole configurations

$$\begin{aligned}
 R &= \sqrt{2r} - \left(\frac{\gamma^2}{4\sqrt{2}r^{3/2}} + \dots \right) + \sqrt{2}\mathcal{P}r^{3/4}e^{-2r}(1 + \frac{1}{r} + \dots) + \mathcal{O}(e^{-3r}), \\
 \phi &= \frac{\phi_\infty}{\sqrt{6}} + \sqrt{\frac{2}{3}}r - \frac{1}{4}\sqrt{\frac{3}{2}}\log r + \left(\frac{5\sqrt{3}\gamma^2}{64\sqrt{2}r^2} + \dots \right) + \sqrt{\frac{3}{2}}\mathcal{P}r^{1/4}e^{-2r}(1 + \frac{1}{2r} + \dots) + \mathcal{O}(e^{-3r}), \\
 \omega &= \frac{\gamma}{\sqrt{r}}(1 + \dots) + \mathcal{C}r^{1/2}e^{-2r}(1 + \dots) + \mathcal{O}(e^{-3r}), \quad \nu = 1 - \frac{K}{2^{5/2}r^{3/4}}e^{-2r+a\phi_\infty}(1 + \dots),
 \end{aligned} \tag{33}$$

where γ , K , \mathcal{C} , ϕ_∞ and \mathcal{P} are free parameters.

The construction of the conserved quantities for this type of asymptotics is an interesting problem. We start by evaluating the on-shell action. By using the scalar and abelian field equations, one can show the volume term in the action (1) reduces to the integral of a total derivative

$$I_B = \frac{1}{4\pi} \int_{\mathcal{M}} d^5x \sqrt{-g} \left(-\frac{1}{3a} \nabla^2 \phi - \partial_\mu (\sqrt{-g} e^{-4a\phi} f^{\mu\nu} W_\nu) \right), \tag{34}$$

and thus can be expressed in terms of surface integrals. However, the contribution of the abelian gauge field in the above expression can easily be proven to vanish. As a result, considering the metric ansatz (22) and Wick rotating $t \rightarrow i\tau$, we find the following expression of the total Euclidean action (where we included also the Hawking-Gibbons boundary term)

$$I = \frac{1}{4\pi}\beta V_3 \lim_{r \rightarrow \infty} \left(e^{2X} (e^{3a\phi} R^3)' + \frac{1}{2} R^3 e^{3a\phi} (e^{2X})' \right) = \frac{1}{8\pi}\beta V_3 \alpha + \frac{1}{4\pi}\beta V_3 \lim_{r \rightarrow \infty} e^{2X} (e^{3a\phi} R^3)', \quad (35)$$

where V_3 is the three-sphere area (note that we used the equation of motion for X in the derivation of the last term). Here β is the periodicity of the time coordinate on the Euclidean section. For regular solutions, β takes arbitrary values (while $\alpha = 0$). The value of β for black hole solutions is fixed by requiring the absence of conical singularities

$$\beta = \frac{4\pi}{\alpha} e^{3a\phi_h} R_h^3. \quad (36)$$

As expected, the total action presents an infrared divergence, as implied by the asymptotic expressions (33). This divergence is associated with the infinite volume of the spacetime manifold.

A common approach - background subtraction, uses a second, reference spacetime to identify divergences which should be subtracted from the action. After subtracting the (divergent) action of the reference background, the resulting action will be finite. This is the method used in previous studies on gauged supergravities with a Liouville-type dilaton potential and nonabelian fields [22], [10]. In these cases, reference spacetime was taken to be the corresponding nonabelian BPS solution. As found in [10], the mass and action of the non-BPS solutions computed in this way generically diverges. However, among all black hole and regular solutions, there are special configurations with finite mass which form a discrete set, corresponding to $\gamma = 0$ in the asymptotic expansion (33).

At a conceptual level, the background subtraction method is not entirely satisfactory, since it relies on the introduction of a spacetime which is auxiliary to the problem. In some cases the choice of reference spacetime is ambiguous -for example NUT-charged solutions (see e.g. the discussion in [23]). This method requires also a complicated matching procedure of some matter fields on the boundary. It would be nice to have a method that is intrinsic to the solutions at hand, instead of one which requires another solution to compare to.

For asymptotically AdS spacetimes, this problem is solved by adding additional surface terms to the theory action [24]. These counterterms are built up with curvature invariants of a boundary $\partial\mathcal{M}$ (which is sent to infinity after the integration) and thus obviously they do not alter the bulk equations of motion. This yields a finite action and mass of the system. The generalization to asymptotically flat case was considered recently in [25] (see also [26]). The boundary counterterm approach in the case of a Liouville-type dilaton potential is discussed in [27], however by assuming a power series decay at infinity which does not cover the asymptotics (33).

For solution with an asymptotics given by (33), we find that by adding to the total Lorentzian action (1) an AdS-like boundary counterterm of the form

$$I_{ct} = -\frac{1}{8\pi} \int_{\partial\mathcal{M}} d^4x \sqrt{-h} \left(\frac{2}{\ell} + \frac{\ell\mathcal{R}}{8} \right), \quad \text{where } \ell = \sqrt{-8V(\phi)} = e^{-a\phi}, \quad (37)$$

the divergence disappears for $\gamma = 0$ solutions, and we arrive at the simple finite expression for the total Euclidean action $I = I_{\text{bulk}} + I_{\text{surf}} + I_{\text{ct}}$

$$I = \frac{1}{8\pi}\beta V_3 \left(3\sqrt{2}e^{\phi_\infty}\mathcal{P} - \frac{1}{2}\alpha \right). \quad (38)$$

Using these counterterms one can also construct a divergence-free stress tensor by defining

$$T_{ab} = \frac{2}{\sqrt{-h}} \frac{\delta I}{\delta h^{ab}} = \frac{1}{8\pi} (K_{ab} - K h_{ab} - \frac{2}{\ell} h_{ab} + \frac{1}{2} \ell E_{ab}), \quad (39)$$

where E_{ab} is the Einstein tensor of the intrinsic metric h_{ab} . The conserved charge associated with time translation is the mass of spacetime, which for $\gamma = 0$ is given by ¹

$$M = \int_{\partial\Sigma} d^3x \sqrt{-h} T_t^t = \frac{1}{8\pi} 3\sqrt{2} V_3 \beta e^{\phi_\infty} \mathcal{P}, \quad (40)$$

which agrees with the results in [10], derived by using the background subtraction approach.

Once we have the renormalized action, standard techniques allows us to calculate the full thermodynamics of the black hole. It can be proven that the entropy of these black holes is one quarter of the event horizon area, as expected. By considering the class of regular stationary metrics forming an ensemble of thermodynamic systems at equilibrium temperature $T = \beta^{-1}$ (see e.g. [23]), and applying the standard formalism one finds

$$S = \beta M - I = \frac{V_3}{16\pi} \alpha \beta = \frac{1}{4} A_h. \quad (41)$$

5 Four dimensional reduction

Apart from Schwarzschild-Tangherlini configuration, the five dimensional Einstein gravity present also other classes of solutions. The simplest one consist in the direct product of Schwarzschild black hole and one extra dimension, describing an uniform black string.

It would be interesting to find the corresponding configurations in the $\mathcal{N} = 4$ Romans model. In this context, we consider first the Kaluza-Klein reduction of the action principle (1) with respect the Killing vector $\partial/\partial x^5$. While the five-dimensional metric has the usual parametrization

$$ds^2 = e^{-2\psi/\sqrt{3}} \gamma_{ij} dx^i dx^j + e^{4\psi/\sqrt{3}} (dx^5 + 2J_i dx^i)^2, \quad (42)$$

(with $i, j = 1, \dots, 4$), to reduce the gauge fields it is convinient to use the ansatz

$$W = u(dx^5 + 2J_i dx^i) + \mathcal{W}_i dx^i, \quad A = \Phi(dx^5 + 2J_i dx^i) + \mathcal{A}_i dx^i, \quad (43)$$

where u and Φ are four dimensional scalars; \mathcal{W}_i , \mathcal{A}_i are gauge fields in $D = 4$ with the coresponding field strength tensors \mathcal{G}_{ij} and \mathcal{F}_{ij} respectively. This ansatz ensures that \mathcal{W}_i and \mathcal{A}_i are invariant under the coordinate transformation $x^5 \rightarrow x^5 + f(x^i)$. The four dimensional action takes a relatively simple form written in terms of the modified field strength

$$\mathcal{G}'_{ij} = \mathcal{G}_{ij} + 2uM_{ij}, \quad \mathcal{F}'_{ij} = \mathcal{F}_{ij} + 2\Phi M_{ij}, \quad (44)$$

where $M_{ij} = \partial_i J_j - \partial_j J_i$.

After Kaluza-Klein reduction, the bulk term in (1) leads to the four dimensional action principle

$$\begin{aligned} I_4 = \frac{1}{4\pi} \int d^4x \sqrt{-\gamma} & \left[\frac{\mathcal{R}}{4} - \frac{1}{2} \nabla_i \psi \nabla^i \psi - \frac{1}{2} \nabla_i \phi \nabla^i \phi - e^{2\sqrt{3}\psi} \frac{1}{4} M_{ij} M^{ij} - e^{2\psi/\sqrt{3}+2a\phi} \frac{1}{4} \mathcal{F}'^I_{ij} \mathcal{F}'^{Iij} \right. \\ & - e^{-4\psi/\sqrt{3}+2a\phi} \frac{1}{4} D_i \Phi^I D^i \Phi^I - e^{2\psi/\sqrt{3}-4a\phi} \frac{1}{4} \mathcal{G}'_{ij} \mathcal{G}'^{ij} - e^{-4\psi/\sqrt{3}-4a\phi} \frac{1}{4} \nabla_i u \nabla^i u \\ & \left. - V(\phi) e^{-2\psi/\sqrt{3}} - \frac{u}{\sqrt{-\gamma}} \epsilon^{ijkl} \mathcal{F}'^I_{ij} \mathcal{F}'^I_{kl} - \frac{4}{\sqrt{-\gamma}} \epsilon^{ijkl} \mathcal{W}_i \mathcal{F}'^I_{jk} D_l \Phi^I \right], \end{aligned} \quad (45)$$

which describes a system with an $SU(2)$ gauge field, two $U(1)$ fields and three scalars coupled to gravity. We expect the existence of monopole and dyon solutions within the above action principle, generalizing the configurations considered in [28], [29], [30].

¹Note that the action and mass of the generic solutions diverges like $e^{c_1 r} r^{-c_2} \gamma^{c_3}$, with c_i positive constants depending on the free parameters which enter (33).

The picture simplifies for a five dimensional ansatz with $A_5 = 0$, $J_i = 0$ and a Liouville potential. In this case, we can consistently set to zero the $D = 5$ abelian field W_μ , and the four dimensional system (45) admits a global symmetry $\psi \rightarrow \psi - \sqrt{2}\epsilon$, $\phi \rightarrow \phi + \epsilon$ [7]. As a result, the following condition

$$\psi = \frac{1}{\sqrt{2}}\phi, \quad (46)$$

can be imposed and we end up with a consistent truncation of the $\mathcal{N} = 4$ $D = 4$ gauged supergravity action [2]

$$I_4 = \frac{1}{4\pi} \int d^4x \sqrt{-\gamma} \left[\frac{\mathcal{R}}{4} - \frac{1}{2} \nabla_i \phi \nabla^i \phi - e^{2\phi} \frac{1}{4} F_{ij}^I F^{Iij} + \frac{1}{8} e^{-2\phi} \right]. \quad (47)$$

We can use this result to find new solutions of the Romans' gauged supergravity with $g_1 = 0$ by uplifting known solutions of the Freedman-Schwarz $\mathcal{N} = 4$ $D = 4$ model. The general $D = 4$ regular solutions (generically non-BPS) will describe $D = 5$ nonabelian vortices; there exist also nonabelian black strings, obtained by uplifting the $D = 4$ black hole solutions discussed in [22]. An interesting case is provided by the topological BPS solutions found in [21]. Together with the $k = 1$ exact solution presented in [1], this gives a class of five-dimensional vortex-type solutions with

$$ds^2 = e^{4\phi/3} (dr^2 + R^2(r)(d\theta^2 + f_k^2(\theta)d\varphi^2) - dt^2) + e^{4\phi/3} (dx^5)^2, \quad (48)$$

$$W_\mu = 0, \quad A = \frac{1}{2} \left(\omega(r)\tau_1 d\theta + \left(\frac{df_k(\theta)}{d\theta} \tau_3 + f_k(\theta)\omega(r)\tau_2 \right) d\varphi \right)$$

and

$$\begin{aligned} k = 1: \quad \omega(r) &= \frac{r}{\sinh r}, \quad R^2(r) = 2r \coth r - \omega^2(r) - 1, \quad e^{2(\phi(r)-\phi_0)} = \frac{\sinh r}{2R(r)}, \\ k = 0: \quad \omega(r) &= e^{-r}, \quad R^2(r) = c - \omega^2(r), \quad e^{2(\phi(r)-\phi_0)} = \frac{e^r}{R(r)}, \\ k = -1: \quad \omega(r) &= \frac{r}{\sinh(r+c)}, \quad R^2(r) = 2r \coth(r+c) - \omega^2(r) - 1, \quad e^{2(\phi(r)-\phi_0)} = \frac{\sinh(r+c)}{R(r)}, \end{aligned} \quad (49)$$

where c and ϕ_0 are arbitrary constants. However, we see that the pathological features of the $k \neq 1$ solutions found in $D = 4$ persist after uplifting them to $D = 5$ and they still presents a naked singularity for some finite value of the radial coordinate.

We close this Section by remarking that new nontrivial $D = 4$ nonabelian solutions can be obtained by adjusting to this case the boosting procedure proposed in [30]. In this approach one starts with a purely magnetic static configuration of the Freedman-Schwarz $\mathcal{N} = 4$ $D = 4$ model (47), and uplift it according to (42), (43) (there we take $J_i = W = \Phi = 0$), finding in this way a vortex-type (or black string) solution of the $D = 5$ Romans' theory. The next step is to boost the $D = 5$ solution in the (x^5, t) plane $x^5 = \cosh \beta z + \sinh \beta \tau$, $t = \sinh \beta z + \cosh \beta \tau$. The dimensional reduction of the boosted five dimensional configurations along the z -direction provides new nontrivial $D = 4$ solutions. However, these configurations do not satisfy the field equations of the Freedman-Schwarz model, presenting a nonvanishing abelian field J_i . Instead, they extremize a truncation of (45) with $W = \Phi^I = 0$. It can easily be proven that the causal structure of the seed solutions is not affected by the boosting procedure [30]. A discussion of these solutions will be presented elsewhere in a more general context.

6 Conclusions

In this paper we have studied some properties of the nonabelian solutions in two versions of $D = 5$, $\mathcal{N} = 4$ gauged supergravity model. In the first version, the dilaton potential have a stationary point, which allows for nonabelian solutions with AdS asymptotics. We have presented numerical arguments for the

existence of both globally regular and black hole solutions. However, we have found that the mass of these configurations generically diverges logarithmically, the origin of this behavior residing in both the dilaton and $SU(2)$ sectors of the theory. While it seems to be possible to remove the divergencies associated with the scalar field, it is less clear how to define a mass for configurations with a nonvanishing gauge field. Note that this divergence seems to be a generic feature of the $D = 5$ nonabelian solutions, originating in the special scaling properties of the YM system in this spacetime dimension. To our knowledge, the only method to obtain $D = 5$ spherically symmetric, finite mass solutions with nonabelian fields consists in adding corrections to the YM lagrangian consisting in higher terms of the YM hierarchy [31, 32].

In the second part of this paper we have studied the basic properties of static, purely magnetic, nonabelian solutions with unusual topology for the case of a Liouville dilaton potential. Our solutions can be regarded as complements of the spherically symmetric configurations discussed in [5, 22]. We have added also one more member to the family of known supersymmetric exact solutions with gravitating nonabelian fields. However, a nonspherical topology of the hypersurfaces $r = \text{const.}$, $t = \text{const.}$ changes drastically the structure and properties of the solutions, leading to singular configurations.

Also, we have proposed to remove the infrared divergencies of the total action and mass by using a counterterm approach which unlike background subtraction, does not require the identification of a reference spacetime. It is important to note that the counterterm action proposed here gives results that are equivalent to what one obtains using the background subtraction method. However, we employ it because it appears to be a more general technique than background subtraction, and it is interesting to explore the range of problems to which it applies.

It may be instructive to give also the corresponding counterterm expression for the $\mathcal{N} = 4$ $D = 4$ Freedman-Schwarz gauged supergravity model. Similar to the $D = 5$ Romans model, the computation presented in the literature in this case makes use of a background subtraction method [22]. In this approach, the $\mathcal{N} = 4$ $D = 4$ Freedman-Schwarz model with a bulk action (47), presents apart from the usual Gibbons-Hawking term a supplementary contribution

$$I_{ct} = -\frac{1}{8\pi} \int_{\partial M} d^3x \sqrt{-h} \left(\frac{1}{2\ell} + \frac{\ell \mathcal{R}}{4} \right), \quad \text{where } \ell = \sqrt{-8V(\phi)} = e^{-\phi}, \quad (50)$$

It can easily be verified that the expressions for the solutions' mass and action obtained by using this approach are similar to those presented in [22].

We close by remarking that, by using the relations in [33], we can uplift all $\mathcal{N} = 4$ $D = 5$ configurations to ten dimensional type IIB supergravity, with the solutions so obtained corresponding to a five-brane wrapped in a nontrivial way.

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